SYNTHESIS OF PITCH ATTITUDE CONTROL SYSTEM FOR AIRCRAFT USING MODEL REFERENCE SELF ADAPTIVE APPROACH

By

AMIT

DEPARTMENT OF AERONAUTICAL ENGINEERING

INDIAN INSTITUTE OF TECHNOLOGY KANPUR

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CERTIFICATE

Certified that this work entitled 'SYNTHESIS OF PITCH ATTITUDE CONTROL SYSTEM FOR AIRCRAFT USING MODEL REFERENCE SELF ADAPTIVE APPROACH' has been carried out under my supervision and that this has not been submitted elsewhere for a degree.

Krishna Kumar

(Krishna Kumar)
Assistant Professor
Department of Aeronautical Engineering
Indian Institute of Technology, Kanpur

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AMIT

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This work is concerned with the problem of variation of aircraft parameters in low level approach and landing caused by ground effects, changing aircraft configuration and gravitational components, etc. Attempt has been made to compensate for these undesirable effects through variable feedback gains using model reference selfadaptive approach. Lyapunov's second method has been used to insure the system stability. The stability derivatives have been estimated to illustrate the application of the above approach in determining the appropriate control laws.

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I. INTRODUCTION

1.1. Preliminary Remarks

The flight envelope of a typical aircraft consists of essentially three phases - take-off, cruise, approach and landing. Of these, most crucial from the point of view of comfort and safety is approach and landing. The performance of an overall low approach system which guides the aircraft during this final phase of the flight depends upon the complex interactions among its various components that include the pilot, various ground based landing aids, the airborne guidance equipment, communication, navigational and guidance channels, the aircraft and the flight control system.

The low approach system may further be divided into two principal functional parts - the measuring subsystem and the control subsystem (Fig.1, Ref.1,2). The measuring subsystem performs both sensing and guidance functions as needed to determine the course to be followed by the aircraft. For this, it measures position, velocity, altitude, flight path angle, etc. using various ground based and airborne landing aids. The control system utilizes this guidance information to determine, develop and apply appropriate forces and moments to the airframe to execute the guidance command. To accomplish this successfully it must compute the control laws taking the guidance command, the aircraft dynamics and the actuator

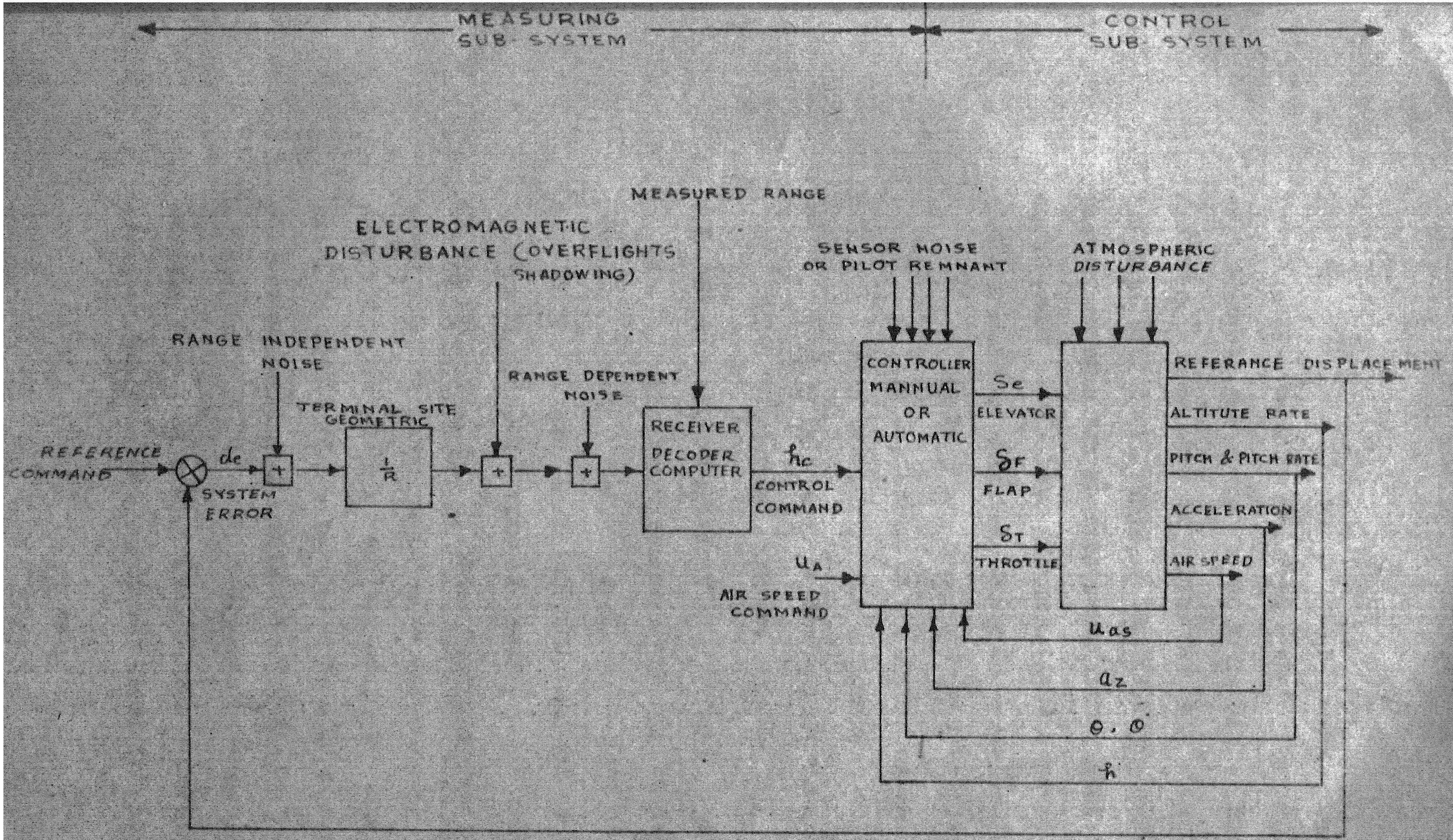


FIG.1 VERTICAL PLANE GUIDANCE SYSTEM

dynamics into account and feed them to the actuator which in turn controls the course to be followed. So far as the aircraft dynamics is concerned it varies not only with the aircraft type but also with whole range of deterministic and random factors the effect of which must be taken into consideration in the development of the appropriate control laws (Ref.1). Some of these factors on which the aircraft dynamics depends are:

(i) Load and load pattern distribution
(ii) Variation in the configuration of the aircraft
(iii) Changes in the environment and the ground effects
(iv) Emergency conditions like engine shutdown or damaged airplane etc.

1.2 Historical Development

The first recorded attempts at designing an automatic landing systems were made in late nineteen forties both at the Wright field in the U.S.A. and at the Blind Landing Experimental Unit of the Royal Aircraft Establishment in the U.K. (Ref. 3,4). These two independent efforts assumed an exponential path for flare to develop differential equations describing the pitching motion. Since the control theory was still in it's infancy, this work heavily depended upon analog simulation studies and flight test data. In 1962, F.J. Ellert of General Electric (USA) reported the development of an automatic landing scheme (Ref. 5) which made it possible to generate the control laws during flare through parametric

optimisation using on board analog computer. He also suggested the need to perform senstivity study for the variation of parameters. In sixties Schoenman of Boeing Company and Doniger of Bendix Corporation (Ref. 6) pointed out the need for compensating the aircraft's dynamic parameter variations during landing and used high gain adaption technique for the purpose. This approach was helpful in making the aircraft's response quite insensitive to the changing dynamic parameters. This method, however, suffered from a major drawback as it not only adversely affected the stability characteristics of aircraft but also required a considerable amount of prior information about system dynamics.

In mid-sixties, the use of digitial computer paved the way for the application of optimal control theory in solving the problem of automatic landing. Different optimal control techniques were employed in conjuction with various performance creteria to generate optimal control laws (Ref. 7, 8, 9). But these methods could not be implemented due to the lack of large, high speed airborne digital computers. In 1971, J.D. Buell (Ref. 10) while investigating the problem of automatic landing found that the results obtained by a simple feedback controller generating an exponential flare path are quite similiar to those obtained by solving two point boundary problem of optimal control. This diminated the need of high speed on board

digital computers for optimal control problem. This study however had only limited objectives and did not account for the variations of aircraft parameters, unpredictable changes in environmental conditions, etc., which must be considered in the design of automatic landing system.

1.3 Purpose and Scope of Investigation.

A brief review of the literature pertinent to the design of automatic landing system reveals that the problem of aircraft parameter variations and it's influence on pitch attitude control has received relatively little attention in the past. Earlier attempts to design automatic landing system, were confined to using higher gains for adaption. This, however, could not account for unforseen changes which often lead to the loss of control. Here, an attempt has been made to synthesize a model reference selfadaptive controller for longitudinal attitude control of the aircraft . Variable feedback gains have been used to adapt to the changes in basic aircraft dynamics, while the variation in control derivatives have to be compensated by changing the effectiveness of control surfaces. Lyapunov's direct method has been used to ensure stability of the aircraft.

Chapter 2 presents the mathematical formulation of the pitching motion using stability axes as the reference frame. A discussion on self adaptive control techniques, presented in Chapter 3 leads to the logical development, using

model reference approach in the synthesis of pitch attitude control system . A suitable Lyapunov's function has been constructed to determine the appropriate control laws for self-adaption.

Finally, to illustrate the application of these results, stability derivatives have been estimated using empirical relations, for a typical 4 engine jet transport aircraft chosen for the purpose.

II. FORMULATION OF THE PROBLEM

The purpose of the low approach system is to regulate the motion of the aircraft during flare. In general this motion can be described by a set of 6 nonlinear coupled differential equations obtained using Newton's laws of motion. Under the simplifying assumptions of linearity, these equations can be decoupled into two separate sets governing the longitudinal and lateral modes of the aircraft motion. With stability axes as the reference frame, the equations of longitudinal motion can be written as (Ref. 33) :

$$\left(\frac{mU}{Sq}\dot{u} - C_{x_u} u\right) + \left(-\frac{c}{2U} C_{x_{\dot{\alpha}}} \dot{\alpha} - C_{x_\alpha} \alpha\right) + \left(-\frac{c}{2U} C_{x_q} \dot{\theta} - C_w (\cos \theta)\theta\right) = \left[C_{F_{x_a}}\right]$$

$$- C_{z_u} u + \left(\left(\frac{mU}{Sq} - \frac{c}{2U} C_{z_{\dot{\alpha}}}\right) \dot{\alpha} - C_{z_\alpha} \alpha\right) + \left(\left(-\frac{mU}{Sq} - \frac{c}{2U} C_{z_q}\right)\dot{\theta} - C_w (\sin \theta)\, \theta\right) = \left[C_{F_{z_a}}\right]$$

$$- C_{m_u} u + \left(-\frac{c}{2U} C_{m_{\dot{\alpha}}} \dot{\alpha} - C_{m_\alpha} \alpha\right) + \left(\frac{I_y}{Sqc} \ddot{\theta} - \frac{c}{2U} C_{m_q} \dot{\theta}\right) = \left[C_{m_a}\right]$$

where, m = Total mass of aircraft

U = Forward velocity of the aircraft in steady flight

S = Wing area

q = Dynamic pressure, $\frac{1}{2} \rho U^2$

u = Perturbation in U

c = Mean aerodynamic chord of the wing

α = Angle of attack

θ = Pitch angle

$\dot{\theta}$ = Pitch rate

$C_{x_u} = \frac{U}{Sq} \frac{\partial F_x}{\partial u}$: Variation of drag and thrust with u

$C_{x_\alpha} = \frac{1}{Sq} \frac{\partial F_x}{\partial \alpha}$: Variation of lift and drag along x-axis

$C_{x_{\dot\alpha}} = \frac{1}{Sq}\left(\frac{2U}{C}\right) \frac{\partial F_x}{\partial \dot\alpha}$: Downwash lag on drag

$C_{x_{\dot\theta}} = \frac{1}{Sq}\left(\frac{2U}{C}\right) \frac{\partial F_x}{\partial \dot\theta}$: Effect of pitch rate on drag

$C_w = -\frac{mg}{Sq}$: Component of gravity

$C_{z_u} = \frac{U}{Sq} \frac{\partial F_z}{\partial u}$: Variation of normal force with u

$C_{z_\alpha} = \frac{1}{Sq} \frac{\partial F_z}{\partial \alpha}$: Slope of normal force curve

$C_{z_{\dot\alpha}} = \frac{1}{Sq}\left(\frac{2U}{C}\right) \frac{\partial F_z}{\partial \dot\alpha}$: Effect of downwash lag on lift of tail

$C_{z_{\dot\theta}} = \frac{1}{Sq}\left(\frac{2U}{C}\right) \frac{\partial F_z}{\partial \dot\theta}$: Effect of pitch rate on lift

$C_{m_u} = \frac{U}{SqC} \frac{\partial M}{\partial u}$: Variation of thrust, slipstream and flexibility with u

$C_{m_\alpha} = \frac{1}{SqC} \frac{\partial M}{\partial \alpha}$: Static longitudinal stability

$C_{m_{\dot\alpha}} = \frac{1}{SqC}\left(\frac{2U}{C}\right) \frac{\partial M}{\partial \dot\alpha}$: Effect of downwash lag on moment

$C_{m_{\dot\theta}} = \frac{1}{SqC}\left(\frac{2U}{C}\right) \frac{\partial M}{\partial \dot\theta}$: Damping in pitch

F_x, F_z = Total force components in the x and z directions respectively

M = Pitching moment

and subscript a refers to the applied forces or moments

The basic non-dimensional stability derivatives used in the design stage can be obtained from various theoritical concepts (Ref. 11 to 19), wind tunnel data, and flight test data. They change continuously as the cruising aircraft makes

a landing approach and flares before touchdown .

2.1 Approximation in Equations of Motion

The mathematical model of the aircraft motion are further modified depending upon the piloting technique used in landing. In one of the piloting techniques, the pitch attitude is mantained constant by proper trimming of the aircraft, while the speed is being suitably varied to control the altitude during landing (Fig. 2). However this method is not suitable for jet aircraft due to sluggish response of the engines to the throttle control. Another technique aims at mantaining a predetermined constant speed using throttle and flaps while the pitch is being continuously varied using elevator to control the altitude (Fig. 2). It being easier to achieve constant speeds, this method has found greater acceptance. Here, in view of the forward speed being mantained constant, the velocity perturbations in U can be equated to zero. Further the first equation governing the force equilibrium in x - direction can be neglected as it essentially governs the speed control only. Thus our problem reduces to synthesis of the pitch altitude control only (Ref. 20). With these assumptions, we need consider only short period dynamics of the aircraft and the equations describing the motion of aircraft in short period mode are

$$\left(\frac{mU}{Sq} s - C_{z_\alpha}\right) \alpha(s) + \left(-\frac{mU}{Sq} s - C_w(\sin\theta)\right)\theta(s) = C_{z_{\delta_e}} \delta_e(s)$$

$$\left(-\frac{c}{2U} C_{m_{\dot\alpha}} s - C_{m_\alpha}\right) \alpha(s) + \left(\frac{I_y}{Sqc} s^2 - \frac{c}{2U} C_{m_q} s\right)\theta(s) = C_{m_{\delta_e}} \delta_e(s)$$

where

$$C_{z_{\delta_e}} = \frac{c}{X_H} C_{m_{\delta_e}}$$

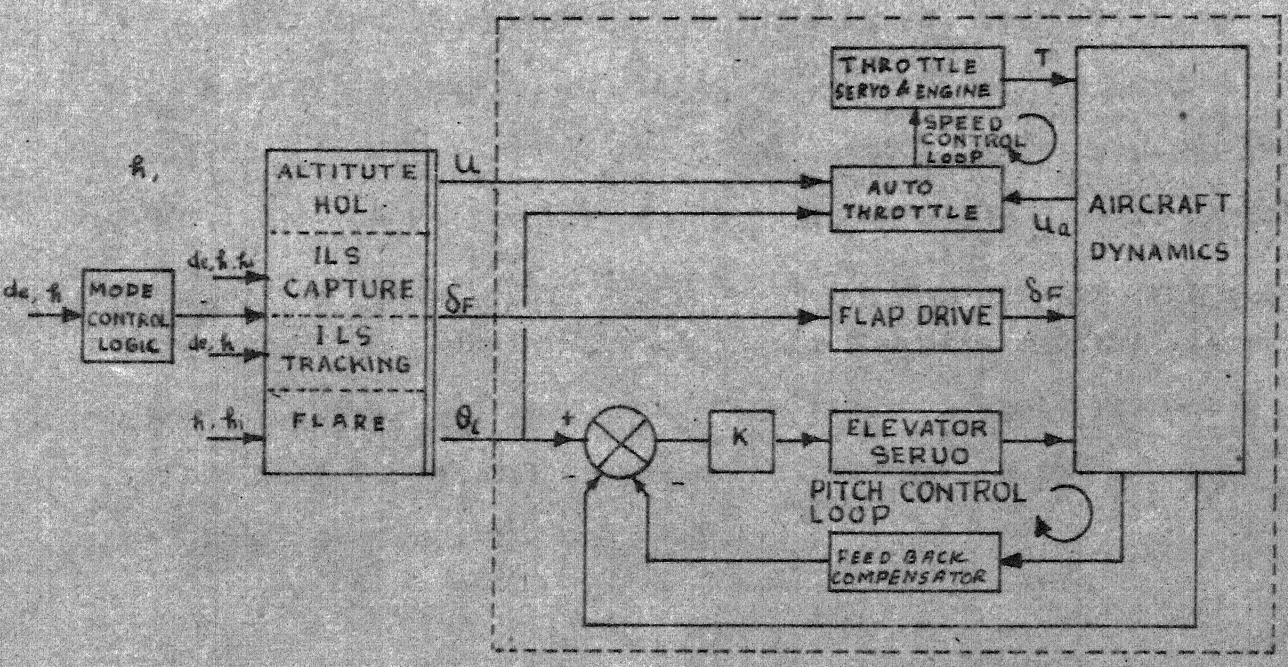


FIG. 2 CONTROL SYSTEM FOR APPROACH

$C_{m_{\delta_e}}$ = Elevator effectiveness

X_H = Distance between aircraft c.g. to the horizontal tail a.c.

δ_e = Elevator deflection

The above modifications in the mathematical model leads to some complications in the design of pitch attitude control system. The operation of the speed control system results in the variation of parameters for pitch attitude control system due to additional deflection of the flaps (Fig. 2). This together with changing aircraft mass and mass centre, ground effects, etc., are responsible for large aircraft parameter variations which must be taken into account in the synthesis of pitch attitude control system.

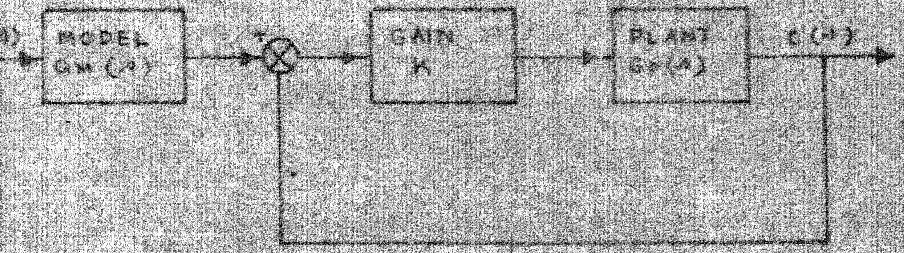
III. SYNTHESIS OF PITCH ATTITUDE CONTROL SYSTEM USING MODEL REFERENCE SELF ADAPTIVE APPROACH

System senstivity to the parameteric variations as discussed in chapter 1 and 2, can be reduced through a feedback control system. However, when the variations are large and a precise control is required, various self adaptive control techniques may be useful for satisfactory response. The different self adaptive control techniques may be divided into three broad classes (Ref. 21) :

1. High Gain Adaption
2. Optimal Adaptive Method
3. Model Reference Method

High gain adaption involves the use of high gains for adaptive action in an attempt to match the prespecified system characteristics. Having the advantage of simplicity, it suffers from a major disadvantage that a considerable amount of prior information about the system dynamics is required and any unforseen changes in the aircraft dynamics may induce system instability due to the high gains (Fig. 3, Ref. 21, 22, 23).

On the other hand, in optimal adaptive control schemes (Ref. 21), the adaptive action is determined with a view to optimize a suitably chosen performance index. In order to solve this optimization problem it is necessary to estimate the states and identify the parameters by studying the instantaneous response of the aircraft under the influence of additional known signals called 'test inputs' (Fig. 3). Since the flare is a precise



TRANSFER FUNCTON FROM S TO C

$$\frac{C(s)}{S(s)} = \frac{KG_P(s)}{1+KG_P(s)}$$

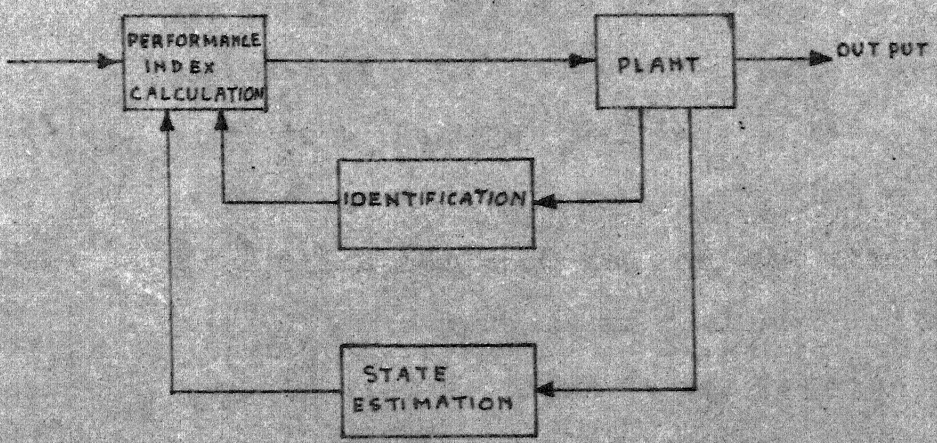
FOR HIGH GAIN, $|KG_P(s)| >> 1$

$$\frac{C(s)}{S(s)} = 1$$

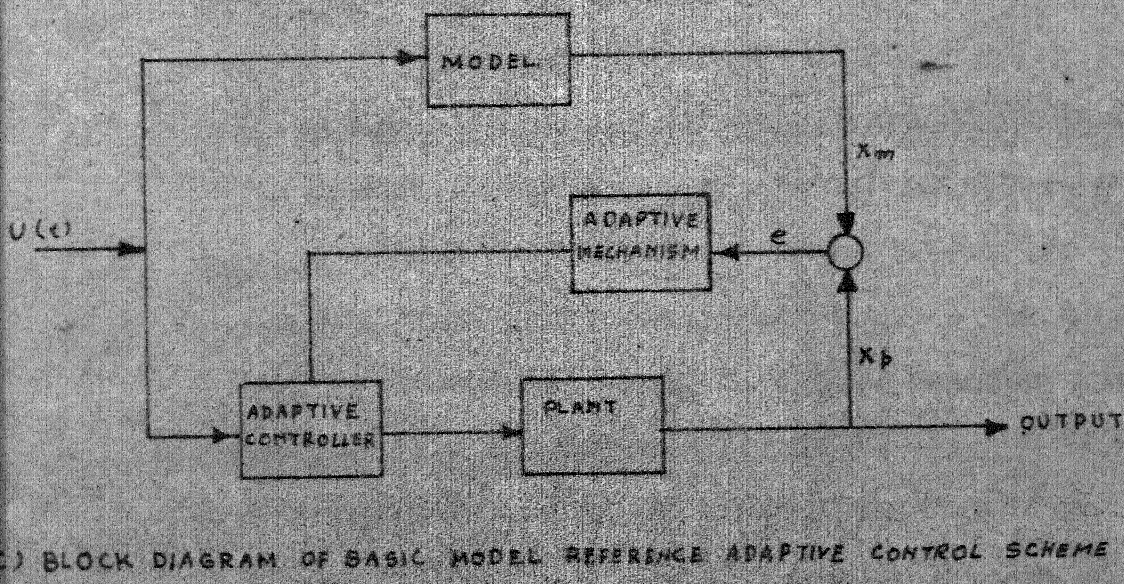
SO, THE TRANSFER FUNCTION FROM THE ACTUAL INPUT R TO THE PLANT OUTPUT C IS

$$\frac{C(s)}{R(s)} = G_M(s)$$

(a) BLOCK DIAGRAM FOR BASIC HIGH GAIN ADAPTIVE MECHANISM.



(b) BLOCK DIAGRAM FOR BASIC OPTIMAL ADAPTIVE CONTROL SCHEME



(c) BLOCK DIAGRAM OF BASIC MODEL REFERENCE ADAPTIVE CONTROL SCHEME

FIG 3 BLOCK DIAGRAM FOR DIFFERENT SELF ADAPTIVE CONTROL SCHEMES.

maneuver, such additional inputs disturbing the aircraft may not be desirable.

This calls for the model reference self adaptive schemes where adaption is based on normal operating inputs to the system and so the additional inputs disturbing the aircraft are not given. In model reference system concept, the changing dynamics of the system is evaluated by comparing the response, with the response of a reference model, (Ref. 21, 24, 25). The reference model is so designed that it's output when excited by the same input command as to the aircraft with varying dynamics provides the desired response. The response error is evaluated by comparing the response of the model and that of the aircraft. This error is the input to the adaptive mechanism that determines the adaption necessary so that the aircraft response closely approximates that of the model (Fig. 3).

3.1 Synthesis of Control System

Considerable amount of information exists concerning the synthesis of model reference self adaptive system. One of the important aspects in the design of such systems is to guarantee stability . Of the various methods, which have been employed to insure stability, Lyapunov's second method has found greater acceptance (Ref. 26 to 32). However, its application in the design of flight control system did not enjoy the same degree of acceptance due to the following reasons:

(i) It requires information about the rate of

change of the aircraft dynamic parameters which can not always be predicted .

(ii) It also requires variations in the basic aircraft dynamic parameters in order to accomplish adaption which in turn may require the use of separate surface flight control system normally not available on the existing aircrafts.

Here, the controller has been synthesized using variable feedback gains for the adaption to the changing basic dynamic parameters of the aircraft of the aerodynamic control surfaces.

3.1.1 Development of Control Laws

The equations of motion for the aircraft with varying parameters and that for the reference model can be written in state variable form as

$$\left.\begin{aligned} \dot{X}_p &= A_p X_p + B_p U \\ \dot{X}_m &= A_m X_m + B_m U \end{aligned}\right\} \quad \ldots\ldots \quad (3.1)$$

where

$X_p = n \times 1$ State vector for aircraft with varying parameters, .

$A_p = n \times n$ System matrix of aircraft whose elements are changing due to the parameter variations.

$B_p = n \times r$ Control matrix for aircraft whose elements are changing due to the variation of control derivatives.

$U = r \times 1$ reference command to be followed, control vector

$X_m = n \times 1$ state vector for reference model

$A_m = n \times n$ system matrix for reference model

$B_m = n \times r$ control matrix for reference model.

In this study, the attention would be confined to the special case of parameter variations in A_p matrix only. The variations in the control matrix B_p have been purposely ignored as they may be adapted by directly regulating the controller effectiveness. With this simplification, B_m can be assumed to be equal to B_p simplifying the state equations to

$$\left.\begin{aligned} \dot{X}_m &= A_m X_m + BU \\ \dot{X}_p &= A_p X_p + BU \end{aligned}\right\} \quad \ldots\ldots (3.2)$$

Incorporating a feedback in the plant, the equations are modified to

$$\left.\begin{aligned} \dot{X}_m &= A_m X_m + BU \\ \dot{X}_p &= (A_p + BF) X_p + BU \end{aligned}\right\} \quad \ldots\ldots (3.3)$$

where

$F = r \times n$ feedback matrix

For the error vector e defined as

$$e = X_m - X_p \qquad \ldots\ldots (3.4)$$

$$\dot{e} = A_m e + (A_m - A_p - BF) X_p \qquad \ldots\ldots (3.5)$$

For adaption to the variation of elements of matrix A, the feedback gains have to be continuously adjusted such that

$$A_m = A_p + BF_t \qquad \ldots\ldots (3.6)$$

where F_t represents the variable feedback gain matrix at instant t. Equation (3.5) then takes the form

$$\dot{e} = A_m e + BF_e X_p \quad \ldots\ldots (3.7)$$

where $\quad F_e = F_t - F = f_{ij}$

Constructing the Lyapunov's function V, as

$$V = \frac{1}{2}\left\{ e^T P e + \sum_i \sum_j f_{ij}^2 \right\} \ldots\ldots \quad (3.8)$$

where P and Q are symmetric positive definite matrices satisfying the criteria for Lyapunov - stability, $A_m^T P + P A_m = - Q$

Differentiating Equation (3.8) with respect to time leads to

$$\dot{V} = -\frac{1}{2} e^T Q e + x_p^T \; F_e^T \; B^T P e + \sum_i \sum_j f_{ij} \; \dot{f}_{ij} \ldots (3.9)$$

Thus for insuring stability, the problem reduces to investigating the condition for which $\dot{V}$ becomes negative definite. Since $-\frac{1}{2} e^T Q e$ is already negative definite the stability would be guaranteed provided

$$x_p^T \; F_e^T \; B^T P e + \sum_i \sum_j \; f_{ij} \; \dot{f}_{ij} \; = 0 \ldots\ldots (3.10)$$

which leads to

$$\dot{F} = B_m^T P e X_p^T \quad \ldots\ldots (3.11)$$

3.2 Illustration

To investigate the problems of parameter variations associated with low level, low speed flying and study the feed-back considerations for approach and landing, a 4 - engined jet transport aircraft belonging to the class of Boeing 707, DC - 8 or CV - 990 was selected. For the given configuration ofthe

airplane (Fig. 4, Table 1) various stability derivatives were estimated using the method given in Appendix A and Appendix B. It was observed that the principal cause of variation of the derivatives from approach to flare was variation of lift curve slope for wing and horizontal tail. The expressions for elements in, matrix A are indicated below

$$a_{11} = \frac{C_{Z\alpha}}{mU/Sq}$$

$$a_{12} = \frac{C_w}{mU/Sq}$$

$$a_{13} = 1$$

$$a_{21} = 0$$

$$a_{22} = 0$$

$$a_{23} = 1$$

$$a_{31} = \frac{S^2\ q^2\ c^2\ C_{m\dot{\alpha}}\ C_{Z\alpha} + 2m\ U^2\ Sqc\ C_{m\alpha}}{2\ m\ U^2\ I_y}$$

$$a_{32} = \frac{S^2\ q^2 c^2}{2m\ U^2\ I_y}\ C_{m_{\dot{\alpha}}}\ C_w$$

$$a_{33} = \frac{S\ q\ c^2\ C_{m_{\dot{\theta}}} + c\dot{S}q\ C_{m_{\dot{\alpha}}}}{2\ I_y\ U}$$

$$b_{11} = \frac{C_{Z_{\delta e}}}{mU/Sq}$$

$$b_{21} = 0$$

$$b_{31} = \frac{S^2\ q^2\ c^2\ C_{m_{\dot{\alpha}}}\ C_{Z_{\delta e}} + 2m\ U^2\ Sqc\ C_{m_{\delta e}}}{2\ m\ U^2\ I_y}$$

Substituting the estimated values of the stability derivatives the A_m and B matrices can be written as :

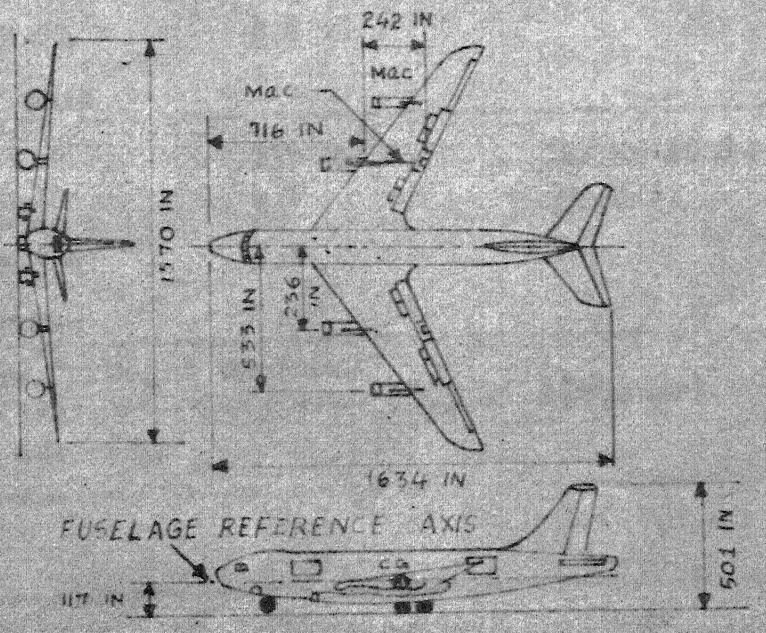


FIG 4 THE AIRCRAFT

TABLE 1

Dimensions and General Data of Airplane

1. **Wing**

Area	2433 ft^2	Mean aerodynamic chord	20.2ft
Sweep (25)	35°	Dihedral	7.0°
Aspect ratio	7.04	Incidence	2.0°
Taper ratio	0.342	Airfoil-root, tip	BAC 313

2. **Horizontal tail**

Area	500 ft^2	Taper ratio	0.447
Sweep (25)	35°	Airfoil	BAC 317
Aspect ratio	3.2		

3. **Vertical tail**

Area	337 ft^2	Taper ratio	0.355
Sweep	31°	Airfoil	BAC 279
Aspect ratio	1.8		

4. **Power plants**

4 JT3O-12 turbojet engines:
S.L. Thrust, take-off, each 12,300 lb(static)

5. **Control surfaces**

Surface	Area (ft^2)	Deflection (deg)	c_f/c
Flap(split)	455.85	50	0.269
Stabilizer	500	+0.5(up)to-14(down)	1.00
Elevator	120.4	+15(down)to-25 (up)	0.25
Outboard aileron	80.4	0(flaps up)	0.105
		±20(flaps 30)	
		±20(flaps 50)	
Inboard aileron	38.8	±18.5(flaps up)	0.145
		±15.7(flaps 30)	0.145
		±17.5(flaps 50)	
Rudder	102.8	±25	0.355

6. **Typical mass characteristics**

Gross weight range	Operating weight empty 106,000 lb	
	Maximum Landing weight 175,000 lb	
	Maximum Taxi weight 250,000 lb	
CG range	15-31 mac	
I_x	2.28×10^6 slug-ft^2	fuselage reference axes
I_Y	3.45×10^6 slug-ft^2	(see 3 view sketch)
I_z	5.75×10^6 slug-ft^2	

TABLE 2

Airplane's Stability Derivatives During Approach and Flare

Stability Derivatives	During approach	During flare just before touchdown
1. α - Derivatives		
C_{L_α}	4.19	5.0
C_{D_α}	0.30	0.50
C_{m_α}	-1.05	-1.25
2. $\dot{\theta}$ - Derivatives		
$C_{L_{\dot{\theta}}}$	6.44	8.5
$C_{D_{\dot{\theta}}}$	Negligible	Negligible
$C_{m_{\dot{\theta}}}$	-12.5	-16.4
3. $\dot{\alpha}$ - Derivatives		
$C_{L_{\dot{\alpha}}}$	-3.0	-3.6
$C_{D_{\dot{\alpha}}}$	Negligible	Negligible
$C_{m_{\dot{\alpha}}}$	-6.5	-8.4

4- u-Derivatives, being small for subsonic speeds, have been neglected.

The unit of angle in these estimations have been taken to be radian.

$$A_m = \begin{bmatrix} -0.76 & 0.11 & 1.0 \\ 0.0 & 0.0 & 1.0 \\ -0.65 & -0.0366 & -0.845 \end{bmatrix}$$

$$B = \begin{bmatrix} -0.036 \\ 0 \\ -0.91 \end{bmatrix}$$

Selecting a positive definite matrix P as

$$P = \begin{bmatrix} 4.31 & 3.12 & -2.94 \\ 3.12 & 3.90 & -2.74 \\ -2.94 & -2.74 & 2.38 \end{bmatrix}$$

The control law obtained by a substitution of the above matrices in Equation (3.11) can be written as

$$\left.\begin{aligned} \dot{f}_{11} &= (2.5\, e_1 + 2.4\, e_2 - 1.1\, e_3)\, x_{p_1} \\ \dot{f}_{12} &= (2.5\, e_1 + 2.4\, e_2 - 1.1\, e_3)\, x_{p_2} \\ \dot{f}_{13} &= (2.5\, e_1 + 2.4\, e_2 - 1.1\, e_3)\, x_{p_3} \end{aligned}\right\} \quad \ldots\ldots (3.12)$$

This offers a rule to determine the variation of the individual elements of the feedback matrix . Needless to say that the Equations (3.12) would have to be integrated using on board analog or digitial computer for supplying the value of matrix F from instant to instant.

IV CONCLUDING REMARKS

In this study, attempt has been made to identify the problem of aircraft parameter variations and emphasize its effect on the pitch altitude control during the crucial phase of approach and flare in landing maneuver. The comparative evaluation of model reference self-adaptive control technique with other control methods like high gain adaption and optimal adaptive technique was undertaken and the advantages of model reference self adaptive systems were emphasized. This model reference approach was used in conjuction with Lyapunov's second method to synthesize the feed back controller. It enables the designer to incorporate engineering specifications directly into the reference model during the initial design stage while the existence of the Lyapunov's function gaurantees the stability of the system. Towards the end, the approach has been illustrated by applying it to the typical transport jet aircraft. Although the synthesis has been carried out with the specific object of control during the approach and flare the same technique can be used for other phases of flight, particularly during the critical maneuvers encountered in military operations.

However, it may be pointed out that in view of severe noise effects near ground, the instrumentation techniques for measuring the state variables in low level, low speed aircraft operations need modifications to achieve the required precision.

In the above study, variations in control derivatives characterising matrix B have not been considered. The difficulty

for taking them into account lies not so much in synthesis but in physical realization. So some alternative approach should be adopted for the purpose. Finally, it would be worth while to investigate hybridization of this model reference approach with optimal adaption.

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APPENDIX A

ESTIMATION OF DRAG POLAR

The evaluation of various stability derivatives associated with the longitudinal motion of the aircraft requires a functional relationship between the lift and drag coefficients called 'drag polar'. The main contributors to the total drag include :

(i) Parasite drag due to skin friction effects.

(ii) Induced drag due to lift.

(iii) Drag due to flaps.

The effect of spoilers, landing gears and control surface deflections has not been considered.

A - 1 Determination of Parasite Drag :

The parasite drag originates from the skin friction of the aircraft surfaces and using empirical relations in conjunction with component build up method, the expression, for this drag coefficient can be written as (Ref. 12)

$$C_{D_o} \simeq \frac{1}{S_{REF}} \Sigma C_{D_\pi} A_\pi$$

where

S_{REF} = 3000 sq. ft. for the aircraft under consideration and the values of C_{D_π} and A_π can be obtained from the following table

TABLE

Component	C_{D_π}	A_π
Wing	.007	$2S$
Fuselage	.0024	$0.75\pi d_f\, l_f$
Nacelles	.006	$\pi\, d_n\, l_n$
Tailplane	.0025	$2\,(S_H + S_V)$

Here

S = Wing Area

d_f = Diameter of fuselage

l_f = Length of fuselage

d_n = Diameter of nacelles

l_n = Length of nacelles

S_H = Horizontal tail area

S_V = Vertical tail area

It may be noted that the subscripts f, n, H and V are used to denote the parameters for fuselage, nacelles, horizontal and vertical tails, respectively.

A - 2 Determination of Induced Drag

The induced drag arises due to the downwash associated with the generation of lift, the empirical relation for it's determination being

$$C_{D_i} = \frac{C_L^{\,2}}{\pi A e}$$

where

C_{D_i} = induced drag

A = aspect ratio of the surface

e = oswald's efficiency factor

C_L = lift coefficient

APPENDIX B

ESTIMATION OF AERODYNAMIC STABILITY DERIVATIVES FOR LONGITUDINAL MOTION

In longitudinal mode of aircraft motion, the aerodynamic forces and moments of interest include lift L , drag D and pitching moment M which vary with changes in angle of attack α , velocity perturbation u, pitch rate $\dot{\theta}$ and vertical acceleration given by $\dot{\alpha}$. It is therefore required to estimate partial derivatives of C_L , C_D , C_m with respect to α , u , $\dot{\theta}$ and $\dot{\alpha}$.

B - 1 α - Derivatives

These derivatives represent the changes in forces and moments due to the change in the angle of attack.

B - 1.1. C_{L_α} , Variation of Lift Coefficient with α

Lift curve slope for the airfoil section, C_{ℓ_α} has strong dependence on the thickness ratio (t/c) as indicated by

$$C_{\ell_\alpha} = 6.28 + 4.7\,(t/c)\,(1+0.00375\,\phi_{te}) \quad /\text{rad.}$$

where

ϕ_{te} = trailing edge angle of the airfoil

It may be pointed out that the effectiveness of this relation is rather limited as it does not take into account influence of several important parameters such as boundary layer, leading edge sharpness, surface roughness and curvature etc. Combining the effect of wing, body and tail, the total C_{L_α} for

the aircraft can be estimated using

$$C_{L_\alpha} = (C_{L_\alpha})_{WB} + (C_{L_\alpha})_H \, \eta_H \frac{S_H}{S} \left(1 - \frac{d\epsilon}{d\alpha}\right)$$

where

$(C_{L_\alpha})_{WB}$ = lift curve slope of the wing body combination

$= K_{WB} \; (C_{L_\alpha})_W$

K_{WB} = correction factor due to flow interference

$= 1 - 0.25\,(d/b)^2 + 0.025\,(d/b)$

$$(C_{L_\alpha})_{W \text{ or } H} = \frac{C_{l_\alpha} A}{\frac{C_{l_\alpha}}{\pi} + \sqrt{\left(\frac{C_{l_\alpha}}{\pi}\right)^2 + \left(\frac{A}{\cos \Lambda_{c/2}}\right)^2}} \; \frac{1}{52.3} \quad \text{per rad.}$$

η_H = 0.9 during approach and flare

$\frac{d\epsilon}{d\alpha}$ = down wash gradient at low speed

$= 4.44 \left(K_A \, K \, K_H \sqrt{\cos \Lambda_{c/4}}\right)^{1.19}$

$c/4$ = sweep angle at quarter chord line

$K_A = \frac{1}{A} - \frac{1}{1 + A^{1.7}}$

$K_\lambda = \frac{10 - 3\lambda}{7}$

$$K_H = \frac{1 - \frac{h_H}{b}}{\sqrt[3]{\frac{2\, l_H}{b}}}$$

h_H = height of tail's root chord plane above wing's root chord plane

l_H = distance between wing's tip aerodynamic centre and the tail's aerodynamic centre.

B - 1.2 C_{D_α}. Variation of Drag Coefficient with α

The drag of the aircraft is given by

$$C_D = C_{D_o} + \frac{C_L^2}{\pi A e}$$

giving

$$C_{D_\alpha} = \frac{\partial C_{D_o}}{\partial \alpha} + \frac{2\, C_L}{\pi A e}\, C_{L_\alpha}$$

where

$\frac{\partial C_{D_o}}{\partial \alpha}$ = variation of profile drag with α, which being small has been neglected.

B - 1.3 C_{m_α}, Variation of Pitching Moment with α

This derivative can be estimated from

$$C_{m_\alpha} = \left(\frac{d\, C_m}{d\, C_L}\right) C_{L_\alpha}$$

where

$\frac{d\, C_m}{d\, C_L}$ = distance between the c.g. and a.c., estimated using the method given in Ref. 11.

B - 2 u - Derivatives

This derivative represents the effect on the forces and moments due to the change in the forward speed while the angle of attack, the elevator angle and the throttle position remain fixed. During the low speed flights such as that during approach and landing, these derivatives can be neglected.

B - 3 $\dot{\theta}$ - Derivatives

These derivatives denote the aerodynamic effects that accompany the rotation of the airplane about a spanwise axis through the c.g. while α remains zero. Here the tail is the main contributor although both wing and tail are affected by

the rotation

B - 3.1 $C_{L_{\dot\theta}}$, Variation of Lift Coefficient with Pitch Rate.

This derivative may be estimated as the sum of wing and tail contributions if small fuselage and nacelles effects are ignored (Ref. 14). Hence

$$C_{L_{\dot\theta}} = (C_{L_{\dot\theta}})_W + (C_{L_{\dot\theta}})_H$$

where

$(C_{L_{\dot\theta}})_W$ = wing contribution to $C_{L_{\dot\theta}}$ in low speed flight

$= \left(\frac{1}{2} + \frac{2\,X_W}{C}\right)(C_{L_\alpha})_W$ per rad.

X_W = rearward distance from airplane's c.g. to wing's a.c.

$(C_{L_{\dot\theta}})_H$ = tail contribution to $C_{L_{\dot\theta}}$

$= 2\,(C_{L_\alpha})_H\,\eta_H\,V_H$ per rad.

V_H = horizontal tail volume coefficient

$= \frac{X_H}{C}\;\frac{S_H}{S}$

X_H = distance between aircraft c.g. to the horizontal tail a.c.

B - 3.2 $C_{D_{\dot\theta}}$, Variation of Drag Coefficient with Pitch Rate

This derivative can be neglected in the low speed subsonic regime of flight.

B - 3.3. $C_{m_{\dot\theta}}$, Variation of Pitching Moment Coefficient with Pitch Rate.

This derivative is contributed by the wing, fuselage and tail, although the fuselage contribution is negligible. So, the expression for $C_{m_{\dot\theta}}$ can be written as

$$C_{m_{\dot{\theta}}} = (C_{m_{\dot{\theta}}})_W + (C_{m_{\dot{\theta}}})_H$$

where

$(C_{m_{\dot{\theta}}})_W$ = wing contribution to $C_{m_{\dot{\theta}}}$

$$= (C_{L_\alpha})_W \cos \quad c/4 \; \frac{A\left(2\left(\frac{X_W}{c}\right)^2 + \frac{1}{2}\left(\frac{X_W}{c}\right)\right)}{A + 2\cos \quad c/4}$$

$$+ \frac{1}{24}\; \frac{A^3 \tan^2 \quad c/4}{A + 6\cos \quad c/4} + \frac{1}{8} \quad \text{per rad.}$$

$(C_{m_{\dot{\theta}}})_H$ = horizontal tail contribution to $C_{m_{\dot{\theta}}}$

$$= -2\, C_{L_{\alpha_H}} \quad {}_H V_H \frac{X_H}{c} \quad \text{per rad.}$$

B - 4 $\dot{\alpha}$ - Derivatives

These derivatives arise from the fact that the pressure distribution on the aerodynamic surface does not adjust instantaneously to its equilibrium value when there is a sudden change in the angle of attack.

B - 4.1 $C_{L_{\dot{\alpha}}}$, Variation in Lift Coefficient with Vertical Acceleration

The wing, fuselage, nacelles and the horizontal tail contribute to the lift due to vertical acceleration, $\dot{\alpha}$, and it can be expressed as (Ref. 15);

$$C_{L_{\dot{\alpha}}} = (C_{L_{\dot{\alpha}}})_W + (C_{L_{\dot{\alpha}}})_f + (C_{L_{\dot{\alpha}}})_n + (C_{L_{\dot{\alpha}}})_H$$

where

$(C_{L_{\dot{\alpha}}})_W$ = wing contribution to $C_{L_{\dot{\alpha}}}$ and is relatively small enough for conventional aircrafts to be neglected.

$(C_{L_{\dot\alpha}})_f$ = fuselage contribution to the $C_{L_{\dot\alpha}}$

$= 2\,(C_{L_\alpha})_f\,\frac{l_f}{c}$

$(C_{L_\alpha})_f$ = lift curve slope for fuselage

$(C_{L_{\dot\alpha}})_n$ = nacelle contribution to the $C_{L_{\dot\alpha}}$

$= 2\,(C_{L_\alpha})_n\,\frac{l_n}{c}$

$(C_{L_\alpha})_n$ = Lift curve slope for nacelles

$(C_{L_\alpha})_H$ = horizontal tail contribution to $C_{L_{\dot\alpha}}$

$= 2\,(C_{L_\alpha})_H\,\eta_H\,V_H\,\frac{d\epsilon}{d\alpha}\,\frac{d\epsilon}{d\alpha}$

For the conventional aircrafts, it has been found that the combined contribution of fuselage and nacelles is about 30% of the contribution by the tail.

B - 4.2 $C_{D_{\dot\alpha}}$, Variation of Drag Coefficient with Vertical Acceleration.

In the low speed subsonic range, associated with approach and flare being considered here, C_D does not vary appreciably with $\dot\alpha$. $C_{D_{\dot\alpha}}$ can therefore be assumed to be negligible.

B - 4.3 $C_{m_{\dot\alpha}}$, Variation of Pitching moment due to Vertical Acceleration.

The principal contributor to this derivative is the horizontal tail.

$C_{m_{\dot{\alpha}}} = (C_{m_{\dot{\alpha}}})_H$, neglecting the relatively small contribution of wing, fuselage and nacelles

where $(C_{m_{\dot{\alpha}}})_H$ = horizontal tail contribution to the $C_{m_{\dot{\alpha}}}$

$$= -2\,(C_{L_\alpha})_H \;\eta_H\; V_H\; \frac{X_H}{C}\; \frac{d\epsilon}{d\alpha}$$

B - 5 Variations in Stability Derivatives

The aircraft's stability derivatives change continuously during landing maneuver due to the changing aircraft configuration, ground effects etc. These effects would now be discussed briefly.

B - 5.1 Effects of Variation in Configuration.

The change in aircraft configuration is caused by flap deflections, operations of landing gears, spoilers, etc. Here, only the effect of flap deflection has been considered. The flap deflection leads to changing aerodynamic characteristics of the wing. An estimate of the resulting change in the lift coefficient is given by (Ref. 13).

$$C_L = C_{L_\alpha}\;(a_\delta)_{C_L}\;\delta\; K_b$$

where

C_L = the increment in lift coefficient due to flap deflection

C_{L_α} = the lift curve slope, per deg.

$(a_\delta)_{C_L}$ = the three dimensional flap effectiveness parameter at constant lift

δ = the flaps deflection, deg.

K_b = the flap span factor

The change in drag coefficient due to the deflection of flaps can be estimated using (Ref. 12).

$$(C_D)_F = (C_{D_o})_F + (C_{D_i})_F + C_{D_{int}}.$$

where

$(C_D)_F$	=	additional drag force due to the flaps deflection
$(C_{D_o})_F$	=	the increase in the parasite drag
	=	$(C_{d_o})_F \left(\frac{S_{WF}}{S} \right)$
$(C_{D_i})_F$	=	the increase in induced drag
	=	$K^2 C_L^2$
$C_{D_{int}}$	=	the interference drag factor
	=	$- 0.15 \, (C_{D_o})_F$
C_d	=	the two dimensional skin friction drag coeff.
S_{W_F}	=	the wing flap reference area
K	=	a constant, 0.21 for the aircraft under consideration.

It should however be noted that the operation of spoilers tends to neutralize the increase in lift due to further deflection of flaps and also result in the increase in drag.

B - 5.2 Ground Effects

The principal ground effects on the airplane are increase in the lift, change in the downwash at the tail and the increase in the lift curve slope which result in significant variations in the stability derivatives.

B – 5.3 Effect of Change in Pitch Attitude.

The changing orientation of the aircraft leads to changes in gravitational force component which in turn affects the aircraft dynamics.

APPENDIX C

DETERMINATION OF MATRIX P

The matrix P appearing in the Lyapunov's function is a nxn- constant real symmetric and its proper determination is very important from the design point of view. Mathematically matrix P can be determined by solving the following matrix equation:

$$A_m^T P + P A_m = - Q$$

where Q is also a nxn constant real symmetric positive definite matrix. So, the determination of matrix P in turn depends upon the proper selection of matrix Q and solution of the above matrix equation.

C - 1 Selection of Matrix Q

The choice of matrix Q depends upon the following performance index to be minimized in this design procedure

$$J(e) = \int_0^{\infty} e^T Q \, e \, dt$$

An acceptable design from a physical view point will be produced by the Lyapunov's design method if and only if matrix Q has been assigned suitable value. Unfortunately, little theory exist on the problem of performance index selection and invariably some trial and error is required.

Considering the system described by a state variable representation

$$\dot{e} = A_m e + BF_e X_p$$

This can be diagonalised, by means of a linear transformation i.e.

$$e = M y$$

where coloums of M are eigenvectors of matrix A_m to a transformed equation

$$\dot{y} = \Lambda y + M^{-1} BF_e X_p$$

The matrix is a diagonal matrix having eigenvalues as the diagonal elements, or closed loop poles of the system. As long as M is nonsingular both setof equations describing the system are completely equivalent. The performance index corresponding to our transformed system of equations is

$$J(y) = \int_0^{\infty} y^* M^* Q M y \, dt = \int_0^{\infty} y^* Q_y y \, dt$$

where * is used to denote the complex conjugate of the transpose.

In terms of this performance index the designer has the option of specifying Q_y . Consideringthe autonomis system

$$\dot{y} = \Lambda y$$

Because is a diagonal matrix, any given y element has no effect on other y elements, i.e., $y_i(t)$ will be of the form

$$y_i(t) = e^{-\lambda_i t} y_i \, (t=0) \quad , i = 1,2, \ldots, n$$

where denote i^{th} eigenvalue. Because of the absence of coupling there is no reason to choose Q_y to be other than the diagonal matrix. For selection of the diagonal elements of Q_y, consider two uncoupled state variables y_i and y_j .

where $|\text{Real}(\lambda_i)| \gg |\text{Real}(\lambda_j)|$. The time constant of y_i will be much smaller than that of y_j in such a case and consequently the contribution of y_i to $J(y)$ will be negligible compared to that of y_j. As a result the intution says that the weighting applied to y_j in the perfomance index should be much larger than the weighting applied to y_i.

C -2 Solution of Matrix Equation $A_m^T P + PA_m = -Q$

This matrix equation can be solved by using well established analytic or numerical techniques (Ref. 34).